Chapter 8 Applications of Straight-Line Graphs ⊕
Reteach Worksheet
8.2 Linear Law

Name: ______________________________ Date: ________________

Class: ________________

Notes

1 The Linear Law describes a linear relation between two variables, say, X and Y. When X and Y are in a linear relation, they are defined by the linear equation $Y = mX + c$, where m is the gradient and c is the Y-intercept of the line.

Example 1

(a) The table shows experimental values of two variables, x and y, which are connected by an equation of the form $xy = ax + b$, where a and b are constants.

x	1.0	2.0	3.0	4.0	5.0	6.0
y	2.77	2.08	1.66	1.38	1.19	1.04

(i) Using a scale of 4 cm to 1 unit on the X-axis and 2 cm to 1 unit on the Y-axis, plot xy against x and draw a straight-line graph.

(ii) Use your graph to estimate the value of a and of b.

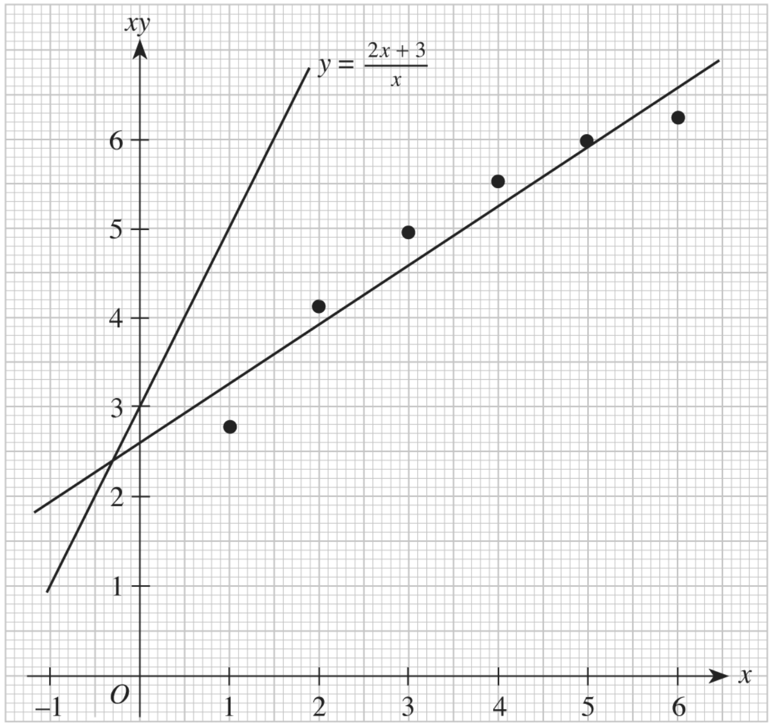
(b) On the same diagram, draw the straight line representing the equation $y = \frac{2x+3}{x}$ and find the value of x for which the two lines intersect.

Solution

(a) **(i)** *Construct a table of values for xy and x.*

X	x	1.0	2.0	3.0	4.0	5.0	6.0
Y	xy	2.77	4.16	4.98	5.52	5.95	6.24

Plot the points and draw the line of closest fit to the data points.



(ii) *Find the y-intercept from the graph.*

The straight line meets the Y-axis at 2.60. So the Y-intercept is 2.60.

Calculate the gradient using any two points on the line of closest fit.

The line passes through (5.5, 6.8) and (1, 3.30).

So the gradient of the line is $\frac{6.8-3.30}{5.5-1}=\frac{7}{9}$.

Form an equation of the form Y = aX + b.

$xy = ax + b$ is in the form $Y = \frac{7}{9}X + 2.6$, where $Y = xy$ and $X = x$.

Compare the equations to obtain the value of a and of b.

Hence $a = \frac{7}{9}$ and $b = 2.6$.

(b) *Express the given expression in linear form, where Y = xy and X = x.*

$$y = \frac{2x+3}{x}$$
$$xy = 2x+3$$
$$Y = 2X+3, \text{ where } Y = xy \text{ and } X = x$$

Plot the straight line on the same axes and identify the point of intersection.

The two graphs intersect at the point where $X = -0.3$.

Find the value of y using the X-coordinate.

Since $X = x = -0.3$,

$Y = 2(-0.3) + 3$
$Y = 2.4$
$y = -8$

Solve.

1A **(a)** The table shows experimental values of two variables, x and y, which are connected by an equation of the form $y = -\frac{ay}{x} + b$, where a and b are constants.

x	0.6	1.2	1.8	2.4	3.0	3.6	6.0
y	2.54	4.05	4.86	5.38	5.96	6.32	7.17

(i) Using a scale of 2 cm to 1 unit on each axis, plot y against $\frac{y}{x}$ and draw a straight-line graph.

(ii) Use your graph to estimate the value of a and of b.

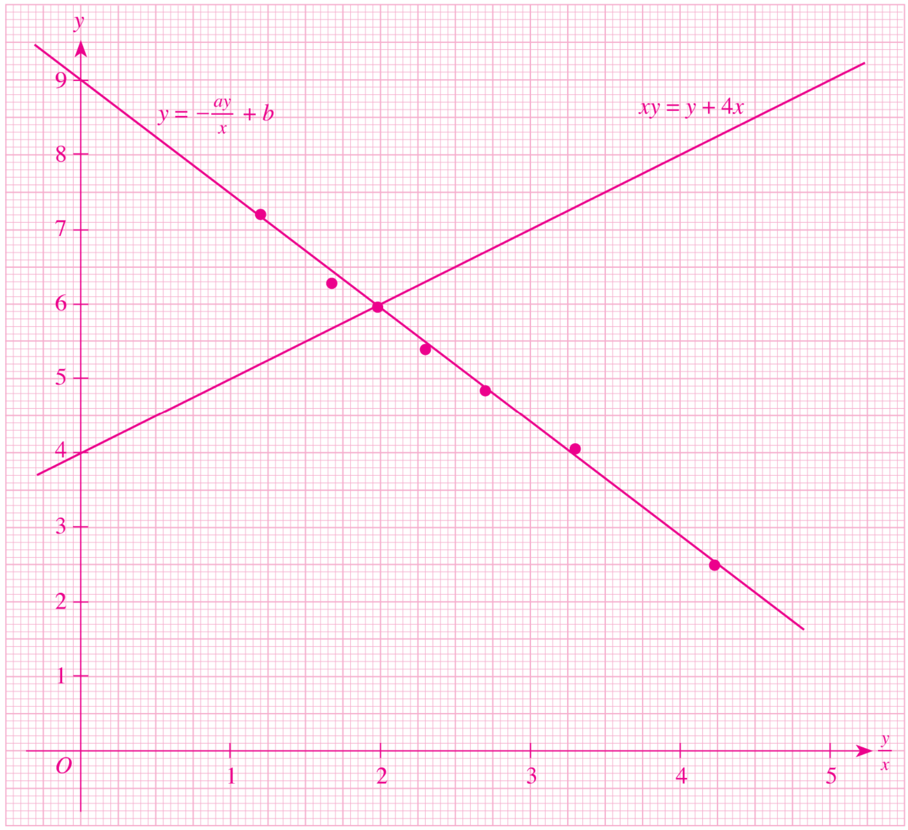
(b) On the same diagram, draw the straight line representing the equation $xy = y + 4x$ and find the value of x for which the two lines intersect.

Solution

(a) **(i)** *Construct a table of values for $\frac{y}{x}$ and y.*

X	$\frac{y}{x}$	4.23	3.30	2.70	2.29	1.99	1.76	1.20
Y	y	2.54	4.05	4.86	5.38	5.96	6.32	7.17

Plot the points and draw the line of closest fit to the data points.



(ii) *Find the Y-intercept from the graph.*

The straight line meets the Y-axis at 9.00. So the Y-intercept is 9.00.

Calculate the gradient using any two points on the line of closest fit.

The line passes through (0.80, 7.80) and (3.60, 3.50).

So the gradient of the line is $\frac{7.80-3.50}{0.80-3.60} \approx -1.54$.

Form an equation of the form Y = aX + b.

$y = -\frac{ay}{x} + b$ is in the form $Y = -1.54X + 9.00$, where $Y = y$ and $X = \frac{y}{x}$.

Compare the equations to obtain the value of a and of b.

Hence $a = 1.54$ and $b = 9.00$.

(b) *Express the given expression in linear form, where Y = y and X =* $\frac{y}{x}$.

$$xy = y + 4x$$
$$\frac{xy}{x} = \frac{y+4x}{x}$$
$$y = \frac{y}{x} + 4$$

$Y = X + 4$, where $Y = y$ and $X = \frac{y}{x}$

Plot the straight line on the same axes and identify the point of intersection.

The two graphs intersect at the point where $Y = 6$.

Find the value of x using the X-coordinate.

Since $Y = y = 6$,

$$y = \frac{y}{x} + 4$$
$$6 = \frac{6}{x} + 4$$
$$2 = \frac{6}{x}$$
$$x = 3$$

1B **(a)** The table shows experimental values of two variables, x and y, which are connected by an equation of the form $\frac{y}{\sqrt{x}} = ax\sqrt{x} + b$, where a and b are constants.

x	0.50	1.00	1.50	2.00	2.50	3.00
y	2.6	4.7	8.2	12.2	17.0	23.2

(i) Using a scale of 2 cm to 1 unit on each axis, plot $\frac{y}{\sqrt{x}}$ against $x\sqrt{x}$ and draw a straight-line graph.

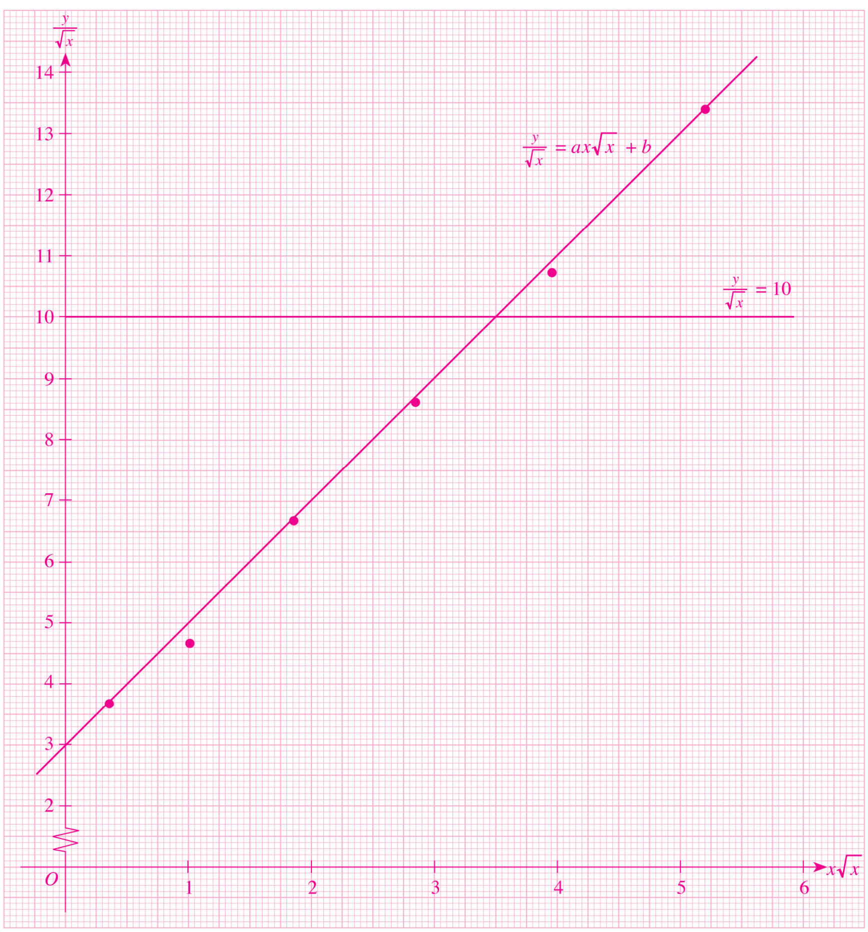
(ii) Use your graph to estimate the value of a and of b.

(b) On the same diagram, draw the straight line representing the equation $2y = 20\sqrt{x}$ and find the value of x for which the two lines intersect.

Solution

(a) **(i)**

X	$x\sqrt{x}$	0.35	1.00	1.85	2.83	3.95	5.20
Y	$\frac{y}{\sqrt{x}}$	3.68	4.70	6.70	8.63	10.75	13.39



(ii) The straight line meets the Y-axis at 3.00. So the Y-intercept is 3.00.

The line passes through (0.50, 4.00) and (5.00, 13.00).

So, the gradient of the line is $\dfrac{13.00-4.00}{5.00-0.50}=2.00$.

$\dfrac{y}{\sqrt{x}}=ax\sqrt{x}+b$ is in the form $Y = 2X + 3$, where $Y=\dfrac{y}{\sqrt{x}}$ and $X = x\sqrt{x}$.

Hence $a = 2$ and $b = 3$.

(b) $2y=20\sqrt{x}$

$\dfrac{2y}{\sqrt{x}}=\dfrac{20\sqrt{x}}{\sqrt{x}}$

$2Y = 20$

$Y=10$, where $Y=\dfrac{y}{\sqrt{x}}$

The two graphs intersect at the point where $X = 3.50$.
Since

$x\sqrt{x}=3.50$

$x^{\frac{3}{2}}=3.50$

$x=(3.50)^{\frac{2}{3}}$

$=2.31$

Example 2

The mass, m grams, of a radioactive substance is given by the formula $m = m_0e^{-kt}$, where t is the time in days after the mass is first recorded and m_0 and k are constants. The table shows experimental values of t and m.

Number of days, t	5	10	15	20	25	30	35	40
Mass, m (grams)	47.9	37.5	32.1	26.3	21.5	17.6	14.4	13.0

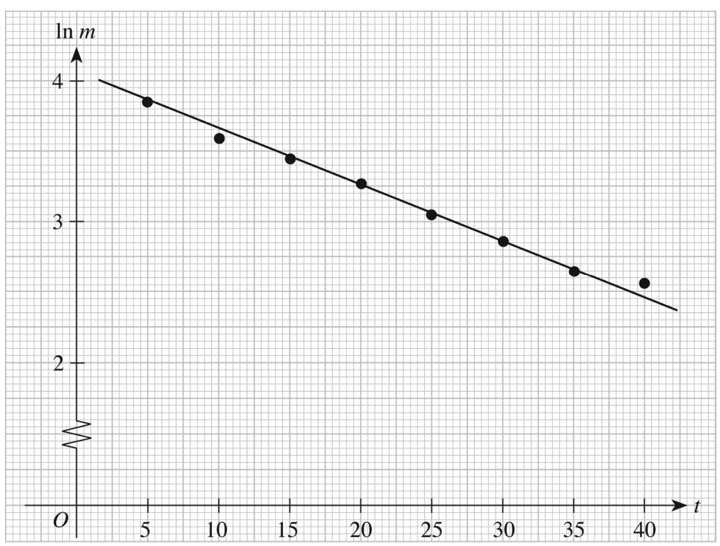
(i) Plot ln m against t for the given data and draw a straight-line graph.
(ii) Use your graph to estimate the value of m_0 and of k.
(iii) Estimate the amount of subtance present after 60 days.

Solution

(i) *Construct a table of values of ln m and t.*

X	t	5	10	15	20	25	30	35	40
Y	ln m	3.87	3.62	3.47	3.27	3.07	2.87	2.67	2.56

(ii) *Plot the points and draw the line of closest fit to the data points.*



Find the Y-intercept from the graph.

The straight line meets the Y-axis at 4.05. So the Y-intercept is 4.05.

Calculate the gradient using any two points on the line of closest fit.

The line passes through (8.0, 3.75) and (28.0, 2.95).

So the gradient of the line is $\frac{2.95-3.75}{28.0-8.0} = -0.04$.

Express the given equation in linear form, where Y = ln m and X = t.

Take logarithms to base e on both sides of the equation $m = m_0e^{-kt}$.

$\ln m = \ln (m_0e^{-kt})$
$\ln m = \ln m_0 + \ln e^{-kt}$
$\ln m = \ln m_0 - kt \ln e$
$\ln m = \ln m_0 - kt$
which is in the form $Y = (-k)X + (\ln m_0)$,
where $Y = \ln m$ and $X = t$.

Equate the gradient and Y-intercept obtained earlier with their respective algebraic representations to obtain the value of m_0 and of k.

Y-intercept $= \ln m_0 = 4.05$ and gradient of line $= -k = -0.04$

Hence $m_0 = e^{4.05} \approx 57.4$ and $k = 0.04$.

(iii) *Substitute the appropriate value of t to obtain the estimate of m.*

After 60 days, $t = 60$.
When $t = 60$, $m = 57.4e^{-0.04(60)} \approx 5.21$.

Hence the amount of substance present after 60 days is 5.21 grams.

Solve.

2A A particle, moving in a certain medium with speed v m/s, experiences a resistance to its motion of R Newtons. It is believed that R and v are related by the equation $R = kv^{\beta}$, where k and β are constants.

Speed, *v* (metres per second)	10	20	30	40	50
Resistance, *R* (Newtons)	102	270	593	900	1344

(i) Plot lg R against lg v and draw a straight-line graph.

(ii) Use your graph to estimate the value of k and of β.

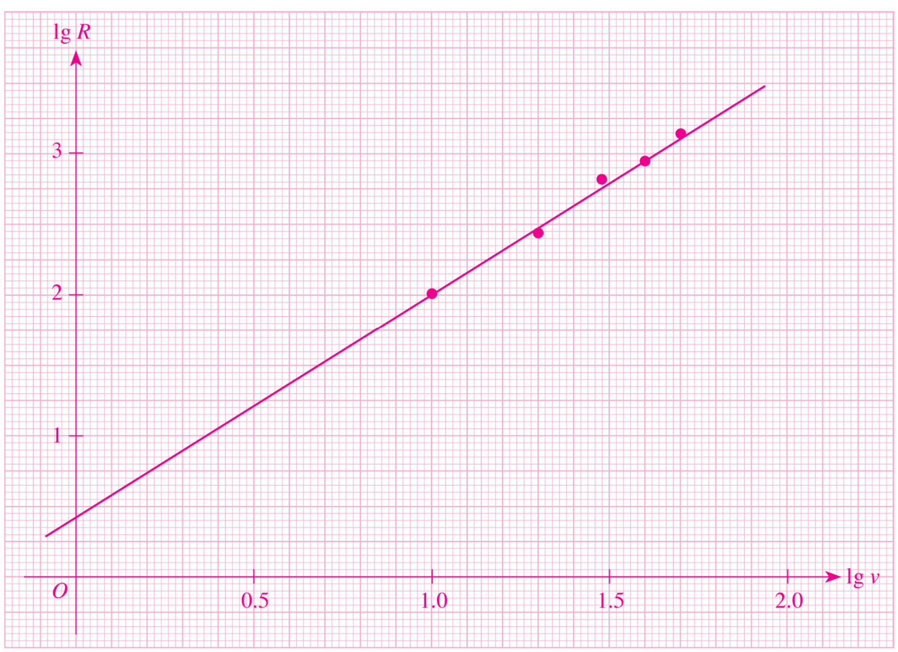
(iii) Estimate the resistance for which the speed is 10 metres per second. Give your answer correct to the nearest hundred.

Solution

(i) *Construct a table of values of lg R and lg v.*

X	lg v	1.00	1.30	1.48	1.60	1.70
Y	lg R	2.01	2.43	2.77	2.95	3.13

Plot the points and draw the line of closest fit to the data points.



Find the Y-intercept from the graph.

The straight line meets the Y-axis at 0.40. So the Y-intercept is 0.40.

Calculate the gradient using any two points on the line of closest fit.

The line passes through (0.9, 1.85) and (1.40, 2.65).

So the gradient of the line is $\dfrac{2.65-1.85}{1.40-0.90}=\dfrac{8}{9}$.

Express the given equation in linear form, where Y = lg R and X = lg v.

Take logarithms to base 10 on both sides of the equation $R = kv^{\beta}$.

$\lg R = \lg (kv^{\beta})$
$\lg R = \lg k + \lg v^{\beta}$
$\lg R = \lg k + \beta \lg v$
which is in the form $Y = \beta X + (\lg k)$,
where $Y = \lg R$ and $X = \lg v$.

Equate the gradient and Y-intercept obtained earlier with their algebraic representations in linear form to obtain the values of k and β.

Y-intercept $= \lg k = 0.40$ and gradient of line $= \beta = \dfrac{8}{9}$.

Hence $k = 10^{0.40} \approx 2.51$ and $\beta = \dfrac{8}{9}$.

(iii) *Substitute the appropriate value of v to obtain the estimate of R.*

When $v = 10$,

$$R = 2.51(10)^{\frac{8}{9}}$$
$$\approx 19.4$$

Hence the resistance for which the speed is 10.0 metres per second is 19.4 Newtons.

2B A controlled experiment was done to estimate the number of fruit flies, N, present in a fly chamber at time t days after the start of the experiment. The results are shown in the table.

Time, t (days)	2	4	6	8	10
Number of fruit flies, N	15	52	120	405	1215

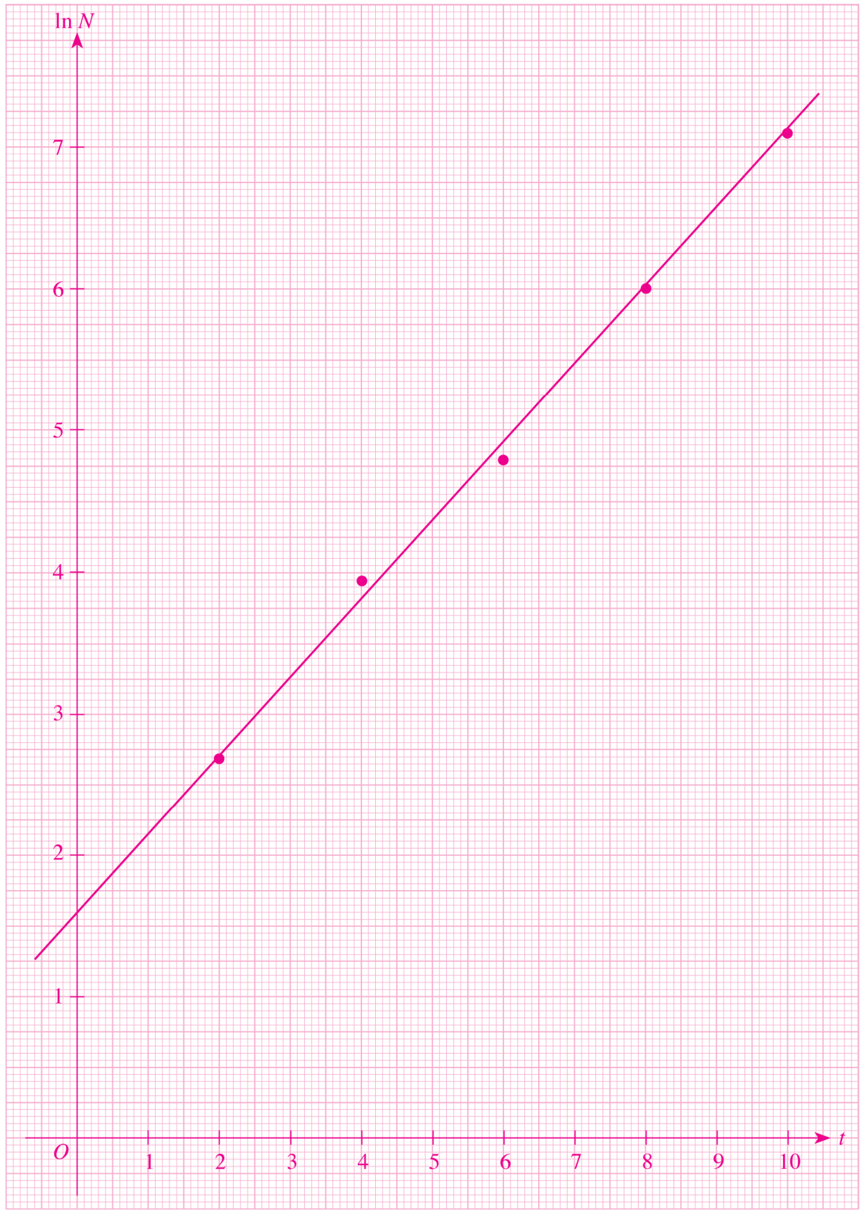
It is known that N and t are related by the equation $N = N_0e^{kt}$, where N_0 and k are constants.

(i) Plot ln N against t and draw a straight-line graph.
(ii) Use your graph to estimate the value of k and of N_0.
(iii) Estimate the number of fruit flies present after 16 days. Give your answer correct to the nearest whole number.

Solution

(i)

X	t	2	4	6	8	10
Y	ln N	2.71	3.95	4.79	6.00	7.10


ln N
7
6
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t

(ii) The straight line meets the Y-axis at 1.60. So the Y-intercept is 1.60.

The line passes through (1, 2.15) and (7.5, 5.75).

So the gradient of the line is $\frac{5.75-2.15}{7.5-1} \approx 0.55$.

Take logarithms to base e on both sides of the equation $N = N_0 e^{kt}$.

$\ln N = \ln (N_0 e^{kt})$
$\ln N = \ln N_0 + \ln e^{kt}$
$\ln N = \ln N_0 + kt \ln e$
$\ln N = \ln N_0 + kt$
which is in the form $Y = (k)X + (\ln N_0)$,
where $Y = \ln N$ and $X = t$.

Y-intercept $= \ln N_0 = 1.60$ and gradient of line $= k = 0.55$.

Hence $N_0 = e^{1.60} \approx 4.95$ and $k = 0.55$.

(iii) When $t = 16$,
$N = 4.95e^{(16)0.55}$
$\approx 32\ 839$

Hence the number of fruit flies present after 16 days is 32 839.

Remark

Round down instead of up since N, the number of flies, is a discrete quantity.